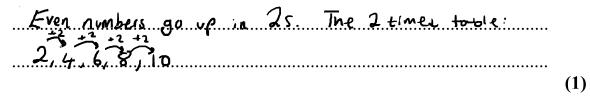
1. The nth even number is 2n.

The next even number after 2n is 2n + 2

(a) Explain why.



(b) Write down an expression, in terms of n, for the next even number after 2n + 2

$$2n+4$$

(c) Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6

$$2n + (2n+2) + (2n+4)$$

$$6n+6$$

$$6(n+1)$$
Because we can factorise out 6, 6n+6 is always

(3) (5 marks) 2. Prove that $(3n+1)^2 - (3n-1)^2$ is a multiple of 4, for all positive integer values of n.

$$(3n+1)(3n+1) - (3n-1)(3n-1)$$

$$(9n^{2}+3n+3n+1) - (9n^{2}-3n-3n+1)$$

$$(9n^{2}+6n+1) - (9n^{2}-6n+1)$$

$$9n^{2}+6n+1 - 9n^{2}+6n-1$$

$$12n$$

$$4(3n)$$

3. Prove, using algebra, that the sum of two consecutive whole numbers is always an odd number.

$$n + n+1$$
 $2n+1$

4. Prove that

$$(2n+3)^2 - (2n-3)^2$$
 is a multiple of 8

for all positive integer values of n.

$$(2n+3)(2n+3) - (2n-3)(2n-3)$$

$$(4n^2 + 6n + 6n + 9) - (4n^2 - 6n - 6n + 9)$$

$$(4n^1 + 12n + 9) - (4n^2 - 12n + 9)$$

$$24n$$

$$8(3n)$$

*5. Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

$$n$$
 and $n+1$ (two consecutive integer)
$$(n+1)^2 - n^2 \qquad (diff. Letween Squares)$$

$$(n+1)(n+1) - n^2$$

$$n^2+n+n+1-n^2$$

$$2n+1$$

Sum of
$$n$$
 and $n+1 = 2n+1$

(4 marks)

6. Prove that $(5n+1)^2 - (5n-1)^2$ is a multiple of 5, for all positive integer values of n.

$$(5n+1)(5n+1) - (5n-1)(5n-1)$$

 $(25n^2+5n+5n+1) - (25n^2-5n-5n+1)$
 $(25n^2+10n+1) - (25n^2-10n+1)$
 $20n$
 $5(4n)$

7. If 2n is always even for all positive integer values of n, prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4.

$$2n \quad \text{and} \quad 2n+2$$

$$(2n)^{2} + (2n+2)^{2}$$

$$4n^{2} + (2n+2)(2n+2)$$

$$4n^{2} + 4n^{2} + 4n + 4n + 4$$

$$8n^{2} + 8n + 4$$

$$4(2n^{2} + 2n + 1)$$

8. Prove that

 $(n+1)^2 - (n-1)^2 + 1$ is always odd for all positive integer values of n.

$$(n+1)(n+1) - (n-1)(n-1) + 1$$

$$(n^{2}+n+n+1) - (n^{2}-n-n+1) + 1$$

$$(n^{2}+2n+1) - (n^{2}-2n+1) + 1$$

$$(n^{2}+2n+1) - (n^{2}+2n-1) + 1$$

$$4n + 1$$

The four times table is always even. An even plus one is odd.

9. Prove algebraically that the sum of the squares of any two consecutive numbers always leaves a remainder of 1 when divided by 4.

(4 marks)